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Seismic instrumentation of high-rise buildings

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Abstract

An optimal assessment method for the design of accelerograph arrays to monitor the seismic response of high-rise buildings is presented. This method uses a finite element model of the structure based on a simplified multi-degree-of-freedom system model defined using the parameter identification method. The off-diagonal element of the Modal Assurance Criterion (MAC) matrix of mode is taken as the target function in order to facilitate the selection of optimum locations. An example for a high-rise building in Dalian indicates that a minimum of 15 locations provide the optimum sites for monitoring the dynamic response of the selected building in view of economic benefit.

Keywords: Accelerograph; High-rise building; Optimal location; Modal assurance criterion

1. Introduction

Earthquakes are the main causes of structural damage and collapse in the world, which then yields huge economic losses and serious casualty. The rapid development of earthquake engineering has accumulated more and more experience on what the structural failure mechanism is and how to make the structures have a better aseismic capability in order to mitigate and reduce earthquake disaster, even some difficulties still need to be solved. A big amount of new structures, such as high-rise buildings, long-span space structures and isolated structures, have been constructed, but their design and seismic resistance should be improved according to realistic monitoring data for the purpose of promoting the development of earthquake engineering.

Some specific regulations and guidances on installing accelerographs have been published in some codes and specifications [1]. It is popular to see distributed accelerographs on dams, nuclear plants and other important engineering sites, but total number is still very limited. To change this situation, the latest "Code for Seismic Design of Buildings" in China requires that structures with a height of over 160, 120 and 80 m and located in the zones of intensity of 7, 8 and 9 should have installed accelerographs [2], which would enable the gradual development of instruments or monitoring arrays on structures. However, it is a fact that it is impossible and unnecessary to install the accelerograph on every floor of structures because of its high cost. The optimal decision should be the most possible way of positioning the accelerograph and monitoring the general seismic response during earthquakes. At present, there are some commonly used optimal methods, such as MKE [3], EI [4], Guyan method [5], GA optimal method and others [6,7]. Reasonable installing principles reflecting the structural vibration characteristics are also important besides choosing suitable optimal methods. So Penny put forward five quantitative principles on how to evaluate different optimal methods on installing

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an accelerograph, i.e., Modal Assurance Criterion (MAC), Mod MAC, SVD ratio, Kinematics' energy ratio and Fisher info matrix [8].

The method of decomposing the MAC optimal function based on the QR row element is adopted in this paper to make optimal distribution on high-rise structures, which is the necessary and valuable improvement of the traditional experience method.

2. Theory

2.1. Structural model simplicity

The main purpose of model simplicity in the case of high-rise structures is to get three simplified matrices, i.e., mass matrix, stiffness matrix and damping matrix. The mass matrix $[M]$ is obtained mainly by concentrating the total mass of each storey on the floor, and not considering the rotation in addition to the horizontal movement.

The stiffness matrix $[K]$ adopts the matrix recognized by the equivalent stiffness parameter method. The total matrix number equals to $m = n(n+1)/2$(n is the freedom number) due to which it is full and symmetry matrix, so at least $(n+1)/2$ group static load should be loaded independently in the finite element model. To make the stiffness solution have enough accuracy, the Least Square Estimation (LSE) was used to recognize the stiffness factors. The detailed process of deducting $[K]$ is as follows:

$$[K] = \begin{bmatrix} k_{11} & k_{12} & k_{13} & \cdots & k_{1n} \\ & k_{22} & k_{23} & \cdots & k_{2n} \\ & & k_{33} & \cdots & k_{3n} \\ & \text{symmetry} & & \ddots & \cdots \\ & & & & k_{nn} \end{bmatrix} = \begin{bmatrix} x_1 & x_2 & x_3 & \cdots & x_n \\ & x_{n+1} & x_{n+2} & \cdots & x_{2n-1} \\ & & x_{2n} & \cdots & x_{3n-3} \\ & \text{symmetry} & & \ddots & \cdots \\ & & & & x_{n(n+1)/2} \end{bmatrix} \quad (1)$$

where

$$\{x\} = \{ x_1 \quad x_2 \quad x_3 \quad \hbar \quad x_{n(n+1)/2} \}^T \quad (2)$$

Rewriting the loading group and displacement into vector, the following is obtained

$$[F] = [F_1 \quad F_2 \quad F_3 \quad \cdots \quad F_m] \quad (3)$$

$$[U] = [U_1 \quad U_2 \quad U_3 \quad \cdots \quad U_m] \quad (4)$$

From

$$[F] = [K] \times [U] \quad (5)$$

the above equation can be written as follows:

$$[u] \times \{x\} = \{f\} \quad (6)$$

where $[u]$ is the extended matrix ($3m \times$ M) from $[U]$ and $\{f\}$ the extended matrix ($3m \times 1$) from $[F]$.

LSE is used to solve the above equation:

$$[u]^T[u]\{x\} = [u]^T\{f\} \quad (7)$$

Let $[u]^T[u] = [A]$ and $[u]^T\{f\} = \{P\}$, then

$$[A]\{x\} = \{P\} \quad (8)$$

where $[A]$ is the symmetry matrix with $M \times M$ rank, $\{P\}$ is the row matrix with $M \times 1$ rank.

The total unknowns $\{x\}$, i.e., stiffness matrix, could be obtained by solving Eq. (8).

The damping matrix is calculated by supposing Rayleigh orthogonal damping, and the damping ratio is 4% for the first two modes:

$$[C] = a[M] + b[K] \quad (9)$$

where

$$a = \frac{2\omega_1\omega_2(\xi_1\omega_2 - \xi_2\omega_1)}{\omega_2^2 - \omega_1^2} \quad (10)$$

$$b = \frac{2(\xi_2\omega_2 - \xi_1\omega_1)}{\omega_2^2 - \omega_1^2} \quad (11)$$

Here, ω_1, ω_2 and ξ_1, ξ_2 are the vibrating frequency and damping ratio of the first and the second vibrating mode, respectively.

2.2. Basic theory of QR

The system response could be expressed in terms of mode overlapping principle as follows:

$$\{u\} = \sum_{i=1}^{s} \phi_i q_i = \Phi_s\{q\} \quad (12)$$

where $\{u\}$ is the physical coordinate, $\{u\} \in R^{s\times 1}$; ϕ_i is the ith mode vector; $\Phi_s \in R^{s\times m}$, the mode; $\Phi_s = [\phi_1, \phi_2, ..., \phi_m]$; q_i the mode coordinate of ith mode; $\{q\} \in R^{m\times 1}$; s represents the accelerograph number; m, the mode number needed to be recognized.

The LSE solution of Eq. (12) is

$$\{\bar{q}\} = \left[\Phi_s^T \Phi_s\right]^{-1} \Phi_s^T \{u\} \quad (13)$$

Considering the measurement noise, this solution could be re-written as follows:

$$\{u\} = \Phi_s\{q\} + \{v\} \quad (14)$$

where v denotes Gaussian white noise with variance of σ^2, covariance of $\{\bar{q}\}$ and $\{q\}$ could be calculated after supposing measured noises are independent of each other and have the same statistical properties for each accelerograph:

$$[P] = E\left[\{q\} - \{\bar{q}\}\right]\left[\{q\} - \{\bar{q}\}\right]^T = \left[\frac{1}{\sigma^2}\Phi_s^T\Phi_s\right]^{-1} = \frac{1}{\sigma^2}[Q]^{-1} \quad (15)$$

where $[Q]$ is Fisher's information matrix. $[P]$ would be the smallest when $[Q]$ has the maximum value, so better estimates could be obtained at this moment. On account of

$$[Q] = \left[\Phi_s^T \Phi_s\right] \tag{16}$$

so that

$$\|Q\| = \left\|\Phi_s^T \Phi_s\right\| = \left\|\Phi_s^T\right\|^2 \tag{17}$$

The above calculating process on $[Q]$ could be treated by selecting suitable Φ_s. It is known from the matrix theory that QR row element decomposition is the simplest and the most effective method to choose the maximum possible solution.

Suppose the obtained mode matrix from finite element model corresponds to the predictable freedom sub-matrix Φ, $\Phi \in R^{n\times m}$, normally $m < n$ and $r(\Phi) = m$. The main row element of Φ^T is going to perform a QR decomposition because it chooses the row vector subset.

$$\Phi^T E = QR = Q\begin{bmatrix} R_{11} & \cdots & R_{1m} & R_{1m+1} & \cdots & R_{1n} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & R_{mm} & R_{mm+1} & \cdots & R_{mn} \end{bmatrix}$$
$$Q \in R^{m\times m},\ \ R \in R^{m\times n},\ \ E \in R^{n\times n},\ \ |R_{11}| > |R_{22}| > \cdots |R_{mm}| \tag{18}$$

where E is the substitution matrix, the corresponding line (i.e., freedom) of $\{\bar{r_1}\}, \{\bar{r_2}\}, \ldots, \{\bar{r_n}\}$ in Φ is the most possible subset ($\{\bar{r_i}\}$ represents the ith row).

2.3. Modal Assurance Criterion (MAC) matrix

It is known from the structural dynamic principles that the structural inherent modes should comprise a group of orthogonal vectors at the nodes, but in fact, the measured mode vector cannot ensure their orthogonality because of the problems of the less measured freedoms than model ones and measuring accuracy limitation. Further, it is even possible to lose many important modes owing to the too small space angles between the vectors. Larger space angles among the measured mode vectors should be guaranteed while choosing measuring points in order to keep the original model properties mostly [9]. Carne thought that the below MAC matrix is a useful tool to evaluate the space angle:

$$\mathrm{MAC}_{ij} = \frac{\left(\Phi_i^T \Phi_j\right)^2}{\left(\Phi_i^T \Phi_i\right)\left(\Phi_j^T \Phi_j\right)} \tag{19}$$

where Φ_i and Φ_j represents the ith and the jth mode vector, respectively.

It is clear from Eq. (19) that the off-diagonal element $\mathrm{MAC}_{ij}(i \neq j)$ in MAC matrix represents the angle between two mode vectors, or in other words, when one off-diagonal element $\mathrm{MAC}_{ij}(i \neq j)$ equals 1, it means that the angle between the ith and the jth vectors is 0, otherwise they are orthogonal while $\mathrm{MAC}_{ij}(i \neq j)$ is zero. So it is desirable to try to make the off-diagonal element in MAC small when selecting the measuring points.

2.4. The procedures for optimal distributing accelerograph

The preliminary scheme for distributing accelerographs over high-rise buildings could be obtained by QR decomposition to mode vector matrix from the finite element analysis. $\Phi(n \times m)$ and $\hat{\Phi}(\hat{n} \times m)$ represent the mode vector matrix formed by measured and left freedoms, respectively, in which, m is the possible or interested mode number, n is the measured freedom number, $\hat{n}$ is the left freedom deducted by the total model measurable freedom from the practical measured freedom.

The MAC matrix elements corresponding to mode i and mode j will be changed as follows once adding the k lines in $\hat{\Phi}$ to Φ:

$$\left(\mathrm{MAC}_{ij}\right)_k = \frac{\left(a_{ij} + \underset{ki}{\hat{\Phi}}\ \underset{kj}{\hat{\Phi}}\right)^2}{\left(a_{ii} + \hat{\Phi}_{ki}^2\right)\left(a_{jj} + \hat{\Phi}_{kj}^2\right)} \tag{20}$$

Φ, $\hat{\Phi}$ and MAC matrix should be modified while adding new accelerographs into the measuring group in order to search for a point in $\hat{\Phi}$ at which the maximum off-diagonal element in MAC matrix could be reduced quickly. In this way, a group of the best solutions that make the biggest off-diagonal element in the MAC matrix the smallest would be obtained after several calculations. But, it is not the case that the optimal effect will be better after the accelerograph number reaches a certain amount because of the slower reducing speed at this stage. Practically, the number of installed accelerographs should be assessed from both economic and optimal aspects according to the following steps:

(1) Calculating the structural mode vector matrix by the finite element method, and then deducing the MAC;
(2) Decomposing the mode vector matrix by QR, using the inferred degree of freedom as the preliminary scheme for distributing the accelerograph, on which the MAC matrix could be found whereas its max off-diagonal element will be deducted further;
(3) Adding one degree of freedom into the left degree of freedom, for example, adding the kth degree of freedom into the measuring one, to get the $(\mathrm{MAC})_k$ matrix, and calculate its maximum off-diagonal element Max_k;
(4) Calculating $f(k) = \mathrm{Max}_k - \mathrm{Max}$, then adding the degree of freedom corresponding to the maximum$|f(k)|(f(k)<0)$ into the distribution scheme; and
(5) Repeating steps (3) and (4) to all left degrees of freedom in MAC.

3. Example

The high-rise building taken as an example is located in the center of Dalian, northeast of China, which has one basement and 80 stories with a height of 339 m. The total area covered is 290,000 m^2. Fig. 1 shows its finite element model for calculation. Its stiffness in both directions is different, so we have simplified the structure as a chain of concentrated masses in weak direction with 81 degrees of freedom as shown in Fig. 2.

The vibration frequencies of the first 10 modes, calculated by the finite element program using the simplified model, are listed in Table 1. Two preliminary positions have been obtained by QR decomposition, i.e., the 9th and the 81st storey, respectively. Fig. 3 shows the three-dimensional diagram of the MAC matrix after the QR preliminary scheme, its maximum off-diagonal value being 0.9641.

The curve of the maximum off-diagonal element of MAC with the measuring point calculated by the Matlab program edited by the authors is presented in Fig. 4. It is easy to see from this figure that the maximum off-diagonal value is only 0.0014 when the measuring point number is 36. But, of course, this is not the optimal decision from the economic view point. Finally, it is considered that point number 15 could be a better choice by trial and error on which the maximum off-diagonal value is 0.0163. Together with the preliminary two points (9 and 81), the other 13 positions are 5, 8, 14, 17, 26, 28, 39, 41, 44, 55, 58, 70 and 71. The final MAC matrix diagram is shown in Fig. 5.

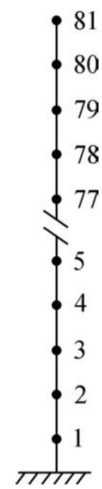

Fig. 2. Simplified model of the structure.

Table 1
Frequencies of the first 10 modes.

Mode	Frequency (Hz)
1	0.17212
2	0.17403
3	0.29088
4	0.51604
5	0.65723
6	0.83024
7	0.91761
8	1.30720
9	1.37650
10	1.38850

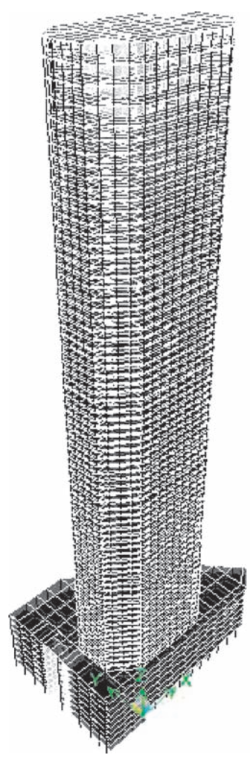

Fig. 1. Finite element model of the structure.

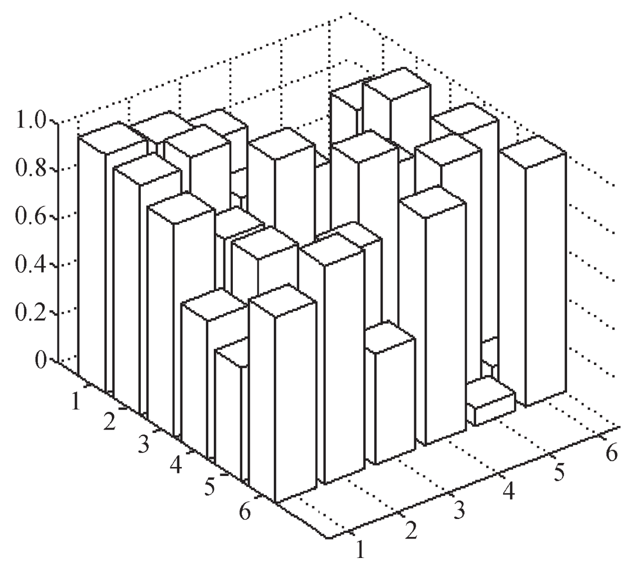

Fig. 3. Three-dimensional diagram of MAC matrix after QR preliminary position.

Besides these points, it has also been decided to increase one point on free field in front of the building to measure the strong ground motion on free field, and also another

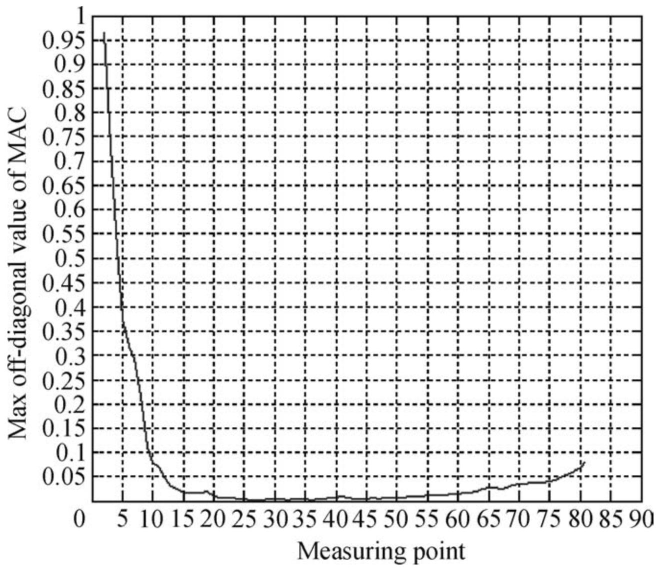

Fig. 4. Variation curve of maximum off-diagonal QR of MAC.

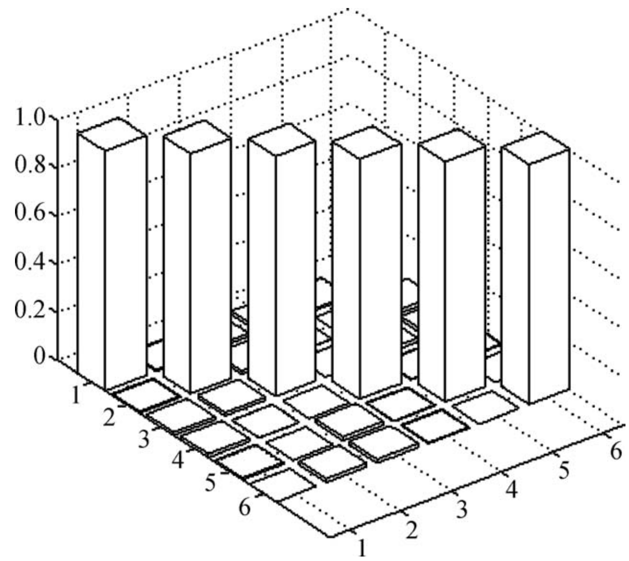

Fig. 5. Three dimensional diagram of MAC matrix using 15 measuring points.

one on the top of building to measure the structural rotation.

4. Conclusions

Depending on the above analysis of optimal assessment of installing accelerographs on high-rise buildings, some conclusions can be summarized as follows:

(1) The distribution of accelerographs over high-rise building can be conducted using a reliable optimal analysis, which could make the final distribution not only economic, but also efficient. This optimal procedure is quite possible and acceptable.
(2) The distribution of accelerographs should be made by consideration of the structural vibration characteristics. Each structure has its own inherent performance during an earthquake and its installing scheme should be chosen according to the optimal decision from the theoretical model analysis.
(3) The QR decomposition method for installing accelerographs on high-rise buildings may be calculated quickly for an ideal structure.

Different optimal procedures have their own characteristics and suitable range for application. Combination of field experience regarding monitoring structural response with results predicted by optimal methods allows structural instrumentation plans to be designed more efficiently.

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